PROBLEM SET 3

It's OK to co-operate with classmates on problem sets. If you get stuck on a problem, don't waste a lot of time on it --- you have better things to do.

The following problems from Starr's *General Equilibrium Theory*, *2nd edition*, are assigned.

14.2

14.6

14.20

24.7

In addition, two problems adapted from past quals are assigned, attached below.

This question is taken from the June 2011 Micro Qual

2. This question focuses on Figure 5.D.3 and the surrounding text taken from Mas-Colell, Whinston, and Green (see the next two pages). The figure represents the case of U-shaped cost curves, often presented in undergraduate microeconomics courses.

   (a) The usual representation of a production function with two inputs, x and y, would be:

   $$q = f(x, y), \quad \frac{\partial f}{\partial x} > 0, \quad \frac{\partial f}{\partial y} > 0, \quad \frac{\partial^2 f}{\partial x^2} \leq 0, \quad \frac{\partial^2 f}{\partial y^2} \leq 0, \quad \frac{\partial^2 f}{\partial x \partial y} > 0.$$

   Is this representation consistent with the situation depicted in Figure 5.D.3? Explain fully.

   (b) Is the situation depicted in Figure 5.D.3 consistent with the existence of general equilibrium in an Arrow-Debreu economy with production? Explain fully.

[First page from Mas-Colell, Whinston, and Green, for use with Problem 2]

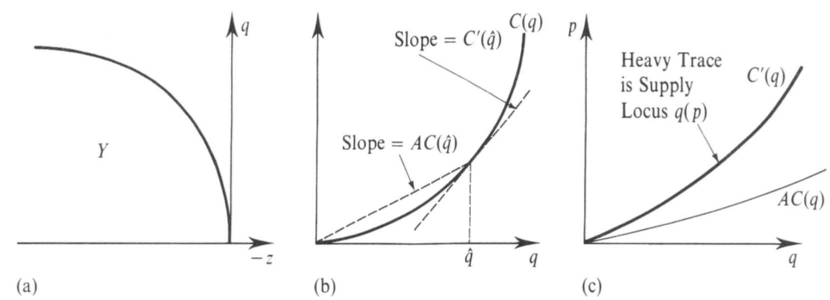

Figure 5.D.1
A strictly convex technology (strictly decreasing returns to scale).
(a) Production set.
(b) Cost function.
(c) Average cost, marginal cost, and supply.

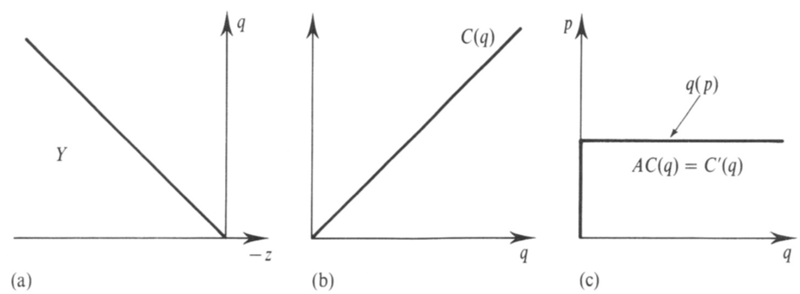

Figure 5.D.2
A constant returns to scale technology.
(a) Production set.
(b) Cost function.
(c) Average cost, marginal cost, and supply.

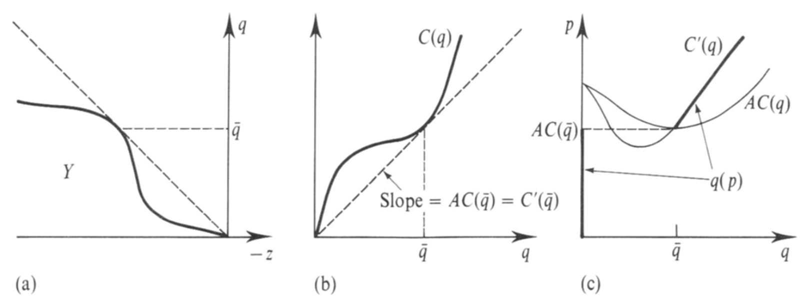

Figure 5.D.3
A nonconvex technology.
(a) Production set.
(b) Cost function.
(c) Average cost, marginal cost, and supply.

(5.D.1) no longer implies that q is profit maximizing. The supply locus will then be only a subset of the set of (q, p) combinations that satisfy (5.D.1).

Figure 5.D.3 depicts a situation with a nonconvex technology. In the figure, we have an initial segment of increasing returns over which the average cost decreases and then a region of decreasing returns over which the average cost increases. The level (or levels) of production corresponding to the minimum average cost is called the *efficient scale*, which, if unique, we denote by $\bar{q}$. Looking at the cost functions in Figure 5.D.3(a) and (b), we see that at $\bar{q}$ we have $AC(\bar{q}) = C'(\bar{q})$. In Exercise 5.D.1, you are asked to establish this fact as a general result.

Exercise 5.D.1: Show that $AC(\bar{q}) = C'(\bar{q})$ at any $\bar{q}$ satisfying $AC(\bar{q}) \leq AC(q)$ for all q. Does this result depend on the differentiability of $C(\cdot)$ everywhere?

The supply locus for this nonconvex example is depicted by the heavy trace in

[Second page from Mas-Colell, Whinston, and Green, for use with Problem 2]

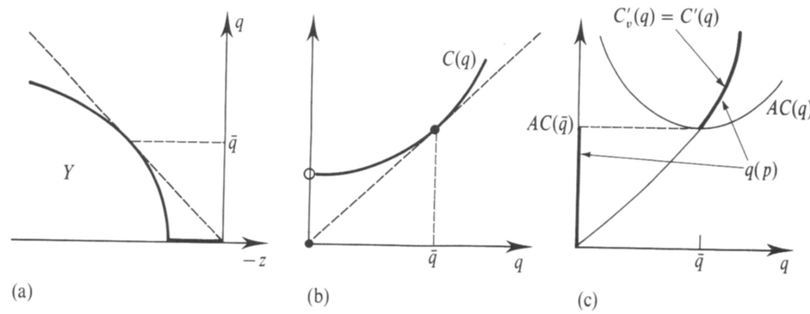

Figure 5.D.4 Strictly convex variable costs with a nonsunk setup cost. (a) Production set. (b) Cost function. (c) Average cost, marginal cost, and supply.

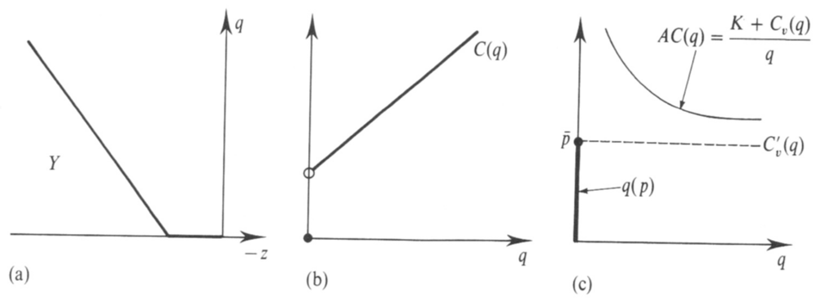

Figure 5.D.5 Constant returns variable costs with a nonsunk setup cost. (a) Production set. (b) Cost function. (c) Average cost, marginal cost, and supply.

Figure 5.D.3(c). When $p > AC(\bar{q})$, the firm maximizes its profit by producing at the unique level of q satisfying $p = C'(q) > AC(q)$. [Note that the firm earns strictly positive profits doing so, exceeding the zero profits earned by setting $q = 0$, which in turn exceed the strictly negative profits earned by choosing any $q > 0$ with $p = C'(q) < AC(q)$.] On the other hand, when $p < AC(\bar{q})$, any $q > 0$ earns strictly negative profits, and so the firm's optimal supply is $q = 0$ [note that $q = 0$ satisfies the necessary first-order condition (5.D.1) because $p < C'(0)$]. When $p = AC(\bar{q})$, the profit-maximizing set of output levels is $\{0, \bar{q}\}$. The supply locus is therefore as shown in Figure 5.D.3(c).

An important source of nonconvexities is fixed setup costs. These may or may not be sunk. Figures 5.D.4 and 5.D.5 (which parallel 5.D.1 and 5.D.2) depict two cases with nonsunk fixed setup costs (so inaction is possible). In these figures, we consider a case in which the firm incurs a fixed cost K if and only if it produces a positive amount of output and otherwise has convex costs. In particular, total cost is of the form $C(0) = 0$, and $C(q) = C_v(q) + K$ for $q > 0$, where $K > 0$ and $C_v(q)$, the *variable cost function*, is convex [and has $C_v(0) = 0$]. Figure 5.D.4 depicts the case in which $C_v(\cdot)$ is strictly convex, whereas $C_v(\cdot)$ is linear in Figure 5.D.5. The supply loci are indicated in the figures. In both illustrations, the firm will produce a positive amount of output only if its profit is sufficient to cover not only its variable costs but also the fixed cost K. You should read the supply locus in Figure 5.D.5(c) as saying that for $p > \bar{p}$, the supply is "infinite," and that $q = 0$ is optimal for $p \leq \bar{p}$.

In Figure 5.D.6, we alter the case studied in Figure 5.D.4 by making the fixed costs sunk, so that $C(0) > 0$. In particular, we now have $C(q) = C_v(q) + K$ for all $q \geq 0$; therefore, the firm must pay K whether or not it produces a positive quantity.

This question is taken from the June 2014 Micro Qual.

3. Consider an exchange economy with n households (consumers) and m goods. Assume that the households have identical preferences. Their preferences are represented by a single utility function $u : \mathbb{R}^m \to \mathbb{R}$ that is strictly increasing, homothetic, and strictly quasiconcave. The latter assumption means that, for consumption bundles x and x' with $u(x) = u(x')$, and for any $\alpha \in (0, 1)$,

$$u(\alpha x + (1 - \alpha)x') > u(x).$$

The households may or may not have different endowments. Let $e^i = (e^i_1, e^i_2, \ldots, e^i_m)$ be the endowment of household i, for $i = 1, 2, \ldots, n$. That is, household i's endowment of good k is e^i_k.

(a) Under what additional conditions (if any) does this economy have a perfectly competitive equilibrium? Is the equilibrium unique? Is the equilibrium efficient?

(b) Under what conditions does this economy have a competitive equilibrium in which no trade occurs?

(c) Suppose the government intervenes in the economy by taxing and redistribution goods before trade takes place. In particular, the government takes a fraction $\beta \in (0, 1)$ of every household's endowment of each good. The government then gives back to the households an equal share of the goods collected. Thus, after taxation and redistribution, household i has the following amount of good k:

$$(1 - \beta)e^i_k + \frac{1}{n}\sum_{j=1}^{n} \beta e^j_k.$$

Then the households trade in the marketplace. Is there a competitive equilibrium? If so, under what conditions is it efficient?

(d) Alternatively, suppose that the government intervenes in a different way. If a household i trades to acquire z^i_1 units of good 1, then the government takes a fraction $\gamma \in [0, 1]$ of these units, so that the household obtains just $(1 - \gamma)z^i_1$ additional units of good 1 in exchange for whatever amounts of the other goods it gives up. The government gives the taxed units of good 1 back to all of the households in equal shares. Assume that the households treat the rebated taxes parametrically (as exogenous). In a competitive equilibrium, how does the price of good 1, relative to the price of each other good, depend on γ?